A Moment Method Analysis for Coplanar Waveguide Discontinuity Inductances

Chien-Wen Chiu and Ruey-Beei Wu

Abstract— **Based on the concept of duality, the quasi-static equivalent inductance of a coplanar waveguide discontinuity is determined from the equivalent capacitance of its complementary structure, i.e., a coplanar strip discontinuity in the free space. For the capacitance calculation, an integral equation governing the excess charge distribution near the discontinuity is solved by the method of moments together with Galerkin's approach. Numerical results for the short end and symmetric step change are presented. The good agreement with the data available from the full-wave analyses reveals that this approach is simple, accurate, and very suitable in the CAD for MMIC.**

I. INTRODUCTION

COPLANAR waveguide discontinuities often appear in the circuit design of monolithic microwave integrated circuit (MMIC), e.g., the short end or open end structure as tuning stub, the symmetric step change as impedance transformer, and so on. At millimeter-wave frequencies, free space radiation and surface wave loss always occur. Some papers based on the full-wave analysis were presented to analyze the short end and open end discontinuities [1]–[4]. It is numerically intensive to apply the full-wave analysis which involves the solution of electromagnetic field and then the frequency dependent scattering parameters. According to several authors' results [2], [3], the CPW discontinuities evidently radiate much less energy than the microstrip discontinuities do. In addition, dispersive effect in the CPW is less obvious than in the other planar transmission lines [5]. Therefore, it is advantageous to apply the quasi-static approximation in the analysis of the CPW discontinuity problem, at least up to the millimeter-wave frequency range.

Some efforts have been made to characterize various CPW discontinuities under the quasi-static approximation. While the published literature provide the equivalent capacitances of CPW discontinuities based on measurements [6] or numerical computations [7], [8], few are available for the equivalent inductance until recently. Bromme *et al.* relied on a unified strip/slot 3-D electromagnetic simulator to investigate the electric properties of various CPW discontinuity structures, but their results lack support from independent evidence [9]. Naghed *et al.* proposed a quasi-static three-dimensional finite difference method to solve the magnetic scalar potential and from which find the magnetic field for the equivalent inductance of the T-junction [10]. The method is very numerically intensive, since it involves the solution for the unknown potential in the three-dimensional space. Also, it can only be applied to the CPW discontinuities which are enclosed by some perfect walls and is inefficient in common cases that the enclosing walls are far from the discontinuities.

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It is conventional to solve the current distribution on the metal strip and then find the equivalent inductance, say for microstrip discontinuities [11]. However, this approach is not suitable here due to the difficulties in the modeling of current distribution in the unbounded metal region of the CPW structures and the incorporation of the continuity of the current distribution near the discontinuity. Instead of directly solving the current distribution, this paper finds the equivalent inductances by employing a duality relation between the inductance of CPW and the capacitance of its complementary configuration, coplanar strip (CPS). For the equivalent capacitance of the CPS discontinuity, we use the concept of excess charge and residual potential proposed by Silvester and Benedek [12]. This makes the numerical computation very efficient since we needs only solve the excess charge in the CPW slot region near the discontinuity. Based on this approach, we calculate the equivalent inductances of CPW short end and symmetric step change. Extensive discontinuity inductances are presented which are useful in the CAD for MMIC. Also, the numerical results for the short end are compared with those obtained by the full-wave analyses [1], [4]. The good agreement verifies that our approach is good for the calculation of the CPW discontinuity inductances.

II. THEORY

Fig. 1 shows a general CPW discontinuity structure, which is formed by a perfectly conducting metallization plane ($y = 0$ plane) above the layered substrates. The metallization plane which is usually assumed to be of negligible thickness is perforated with slots to form the CPW discontinuity. As far as the quasi-static inductance is concerned, it is desired to find the magnetic field due to the current flow in the metallization plane. In common cases that the substrates are nonmagnetic ($\mu = \mu_0$), the presence of the dielectric layers can be neglected since it does not affect the magnetic field and the desired inductance problem in quasi-static approximation [11]. It is sufficient to consider only the space above the plane from the consideration of the symmetry.

The conducting metallization plane shown in Fig. 1 can be divided into two regions, one is the slot region denoted by S and the remaining metal region denoted by $\overline{S}$. From

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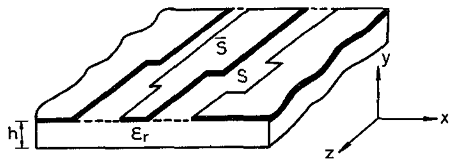

Fig. 1. A general CPW discontinuity structure for inductance computation.

Maxwell's equations, the magnetic field intensity in the upper space satisfies the partial differential equations (PDE)

$$\nabla \times \vec{H} = 0 \quad \text{and} \quad \nabla \cdot \mu_0 \vec{H} = 0 \,. \tag{1}$$

The boundary conditions (BC) at the $y = 0$ plane are such that

$$\hat{n} \times \vec{H} = 0 \quad \text{on} \quad S \quad \text{and} \quad \hat{n} \cdot \vec{H} = 0 \quad \text{on} \quad \overline{S} \tag{2}$$

where $\hat{n} = -\hat{y}$ is the unit vector outward normal to the space $y > 0$. Here, the first condition follows from the symmetry consideration while the second one is due to the fact that the normal component of magnetic field intensity is zero at the surface of a perfect conductor.

On the other hand, let us consider the complementary CPS structure which is formed from Fig. 1 by interchanging the metal and slot regions and replacing the substrate with free space. Similarly, it can be readily obtained from Maxwell's equations that the electric field intensity in the upper space satisfies PDE

$$\nabla \times \vec{E} = 0 \quad \text{and} \quad \nabla \cdot \epsilon_0 \vec{E} = 0 \tag{3}$$

subject to the boundary conditions

$$\hat{n} \times \vec{E} = 0 \quad \text{on} \quad S \quad \text{and} \quad \hat{n} \cdot \vec{E} = 0 \quad \text{on} \quad \overline{S} \tag{4}$$

at the $y = 0$ plane.

It is apparent that (1) and (2) are in mathematical analogy with (3) and (4). Therefore, the magnetic field in a CPW structure is the same as the electric field in its complementary CPS structure, except for a constant. From the concept of duality and the definitions of the capacitance and inductance, we can find a relation between the inductance of the CPW and the capacitance of its complementary CPS, i.e.,

$$L_{\mathrm{CPW}} = \frac{1}{4} \frac{\mu_0}{\epsilon_0} C_{\mathrm{CPS}}(\epsilon_r = 1) \,. \tag{5}$$

It is interesting to compare (5) to the well-known formula proposed by Getsinger [13], i.e.,

$$L_{\mathrm{CPW}} = \frac{1}{4} \frac{\mu_0}{\epsilon_0 \epsilon_{\mathrm{eff}}} C_{\mathrm{CPS}} \,. \tag{6}$$

In case of two dimensional transmission lines, the two formulas are equivalent since by definition

$$\epsilon_{\mathrm{eff}} = C_{\mathrm{CPS}} / C_{\mathrm{CPS}}(\epsilon_r = 1) \tag{7}$$

where C_{CPS} is the capacitance of CPS per unit length. However, (6) becomes confusing for three-dimensional structures since thence the ratio between C_{CPS} and $C_{\mathrm{CPS}}(\epsilon_r = 1)$ is in general different from the conventional ϵ_{eff} which is defined in the two-dimensional case.

Note that the metal region in the complementary CPS structure, i.e., region S, is much easier to handle in the numerical solution than the original metal region $\overline{S}$ in the CPW structure. Hence, it is advantageous to calculate the capacitance of the complementary CPS structure $C_{\mathrm{CPS}}(\epsilon_r = 1)$ and then from which find the desired L_{CPW} by (5). For example, consider the short ended CPW discontinuity which could be modeled as an equivalent inductance of L^e_{CPW}. From above, it is mathematically dual to solve the equivalent capacitance C^e_{CPS} of its complementary open ended CPS discontinuity.

It is not difficult to calculate the capacitances of three-dimensional CPS structures by applying the method of moments [14]. However, to extract the equivalent capacitances of CPS discontinuities more successfully, we employ the approach which was originally proposed by Silvester and Benedek to evaluate the equivalent capacitances of microstrip discontinuities [12]. In this approach, an integral equation is derived for the "excess charge distribution" $\sigma^e(\vec{r}\,')$ on the metal surface S, i.e.,

$$\phi_{\mathrm{res}}(\vec{r}) = \int_S G(\vec{r}, \vec{r}\,') \sigma^e(\vec{r}\,') \, d\vec{r}\,' \tag{8}$$

where $G(\vec{r}, \vec{r}\,')$ is the associated Green's function and $\phi_{\mathrm{res}}(\vec{r})$ is the "residual potential." The integral equation is subsequently solved by applying the method of moments together with Galerkin's approach. Integrating the excess charge yields the desired equivalent discontinuity capacitances C^e_{CPS}. Details about this approach can be found in [12].

III. Numerical Analysis and Results

To begin with, we need to solve the two-dimensional charge distribution for a uniform CPS transmission line. Dividing the conductor strip of CPS uniformly into N segments along the slot width w and using the pulse function as the basis, we can obtain a good approximation for the unknown charge distribution by applying the method of moments. Integrating the obtained charge distribution, we can calculate the two-dimensional capacitance C^{2d}_{CPS} and then the characteristic impedance. It is found that satisfactory convergence can be achieved using $N = 12$. The calculated results, although not presented here, are found to be in good agreement with those obtained by the closed form formula in [15].

Given the two-dimensional charge distribution, the method of moments is again employed to solve the excess charge distribution. Since the excess charge distribution decays fast as far from the discontinuity, we truncate the domain of concern after a certain distance l. In the numerical solution, we use the subdomain square pulse function as the basis and the domain of concern is divided into $M \times N$ cells where M is the numbers of division along the length l. Naturally, if we increase the length l and the number of division $M \times N$, the calculated results will be more accurate, but requiring longer computation time as a tradeoff. After extensive numerical experiments, we choose $N = 12$ and approximately $l = 10w$ in the following computations, which is sufficient to achieve good convergence with smaller than 1% error for the equivalent capacitance C^e_{CPS}.

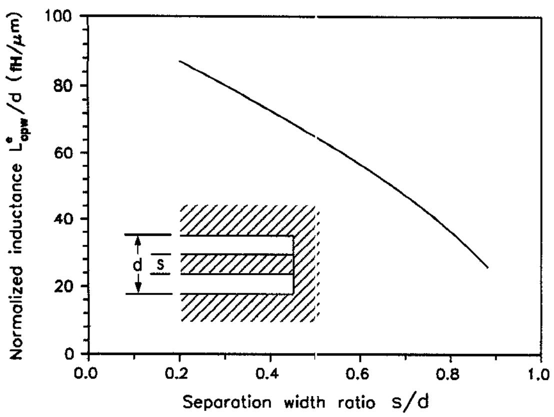

Fig. 2. Equivalent CPW short-end inductance normalized to total line width d versus the separation width ratio s/d.

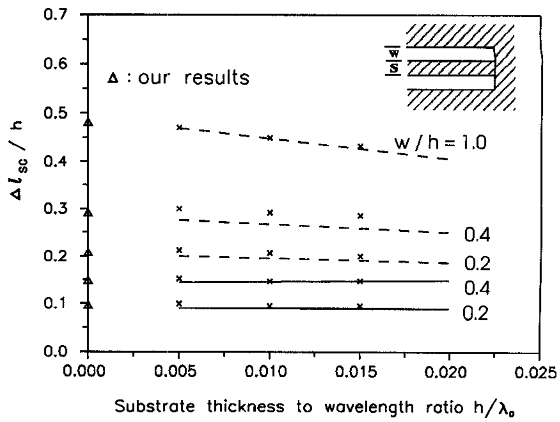

Fig. 3. Calculated effective length extension of shorted CPW by quasi-static approximation as compared to those by the full-wave analyses from [1], [4]. ($h = 0.635$ mm, $\epsilon_r = 9.7$, × × ×× data from [1], ——— $s/h = 0.2$, – – – – $s/h = 1.0$ data from [4].)

The equivalent inductance of the symmetric CPW short end is a function of the slot width w and separation s but is independent of the substrate. Fig. 2 shows the characteristic curve of the CPW short-end inductance normalized by the total line width d versus the ratio s/d. For a fixed total line width d, the equivalent inductance becomes larger for smaller separation s due to a denser current density on the central conductor. Recently, Beilenhoff *et al.* investigated coplanar MMIC's by means of a finite difference method in frequency domain and reported that the finite metallization thickness has a noticeable influence on the short-end circuit behavior [16]. Therefore, the characteristic curve obtained by our analysis which is based on the assumption of zero strip thickness may be subject to some errors in the MMIC design where the strip thickness is not negligible. Nonetheless, the equivalent inductance is about several pH in typical MMIC configurations where the total line width is in the order of 100 μm.

Given the quasi-static short-end inductance L^e_{CPW}, we can calculate the effective length extension $\Delta l_{sc} = L^e_{\text{CPW}}/L^{2d}_{\text{CPW}}$ where the transmission line inductance L^{2d}_{CPW} could be found from C^{2d}_{CPS} by (5). Fig. 3 shows the resultant length extension normalized with respect to $h = 0.635$ mm for several different widths w and separations s. Also shown in the figure for comparison are the results obtained by the full-wave analyses [1], [4] versus substrate thickness to wavelength ratio h/λ_0. It is found that our quasi-static approximation is satisfactory. Actually, the results by full-wave analyses reveal that the short-end inductance has a negligible frequency dependence except for larger w/h at higher frequencies. For typical CPW structure, the applicable frequency range of the quasi-static approximation is up to about 30 GHz.

From the duality between the electric field and magnetic field, the current distribution on the metallization plane of CPW can be found from the tangential electric field components on that of the complementary CPS. Fig. 4 plots the total current distribution on the metal region near the discontinuity for the CPW short end with $s/w = 2.5$. Due to the structure symmetry, only the distribution on one half region is shown here. This figure clearly demonstrates that most of the current flocks round the slot boundary. In the far region, say $3w$ away from the short end, the current is very small such that the housing wall if any present there will have a negligible effect on the equivalent inductance.

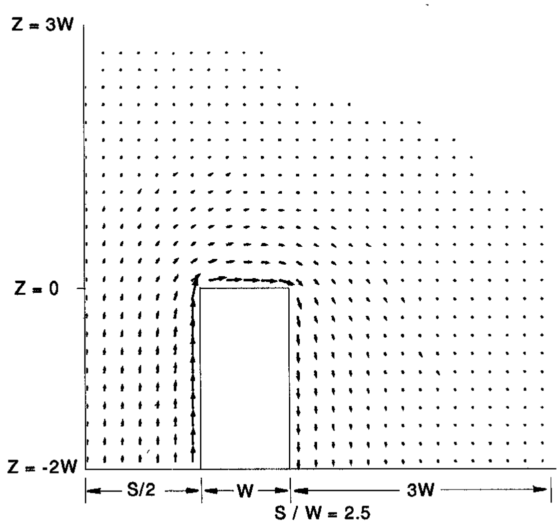

Fig. 4. Current distribution on the metal region near the CPW short end with $s/w = 2.5$.

Following the same approach, we can deal with other CPW discontinuities. Fig. 5(a) shows the equivalent inductances of CPW discontinuity with symmetric step change in width. The total inductance is based on the two port network model given in Fig. 5(b). As shown in Fig. 5(a), the equivalent inductance is about one order of magnitude smaller than the short-end inductance for typical MMIC configurations. Also, the equivalent inductance gradually increases as the step change ratio s_1/s_2 increases. It is worth noting that the effect of the series inductance is not negligible as compared to that of the shunt capacitance in the equivalent circuit of Fig. 5(b). Some previous literature modeled the step change by a lumped shunt capacitance only and extracted the capacitance from the scattering coefficients [6], [8]. Due to the negligence of equivalent inductance, the obtained results e.g., [6, Fig. 8], show an unreasonably irregular tendency and are perhaps questionable.

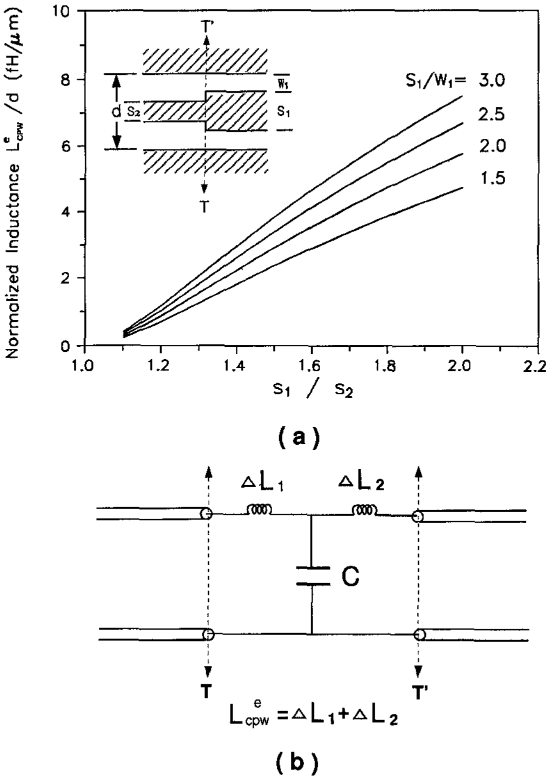

Fig. 5. (a) Calculated equivalent inductance of CPW step change normalized to line width d versus step width ratio s_1/s_2 with different s_1/w_1 as parameter. (b) Equivalent circuit model. The calculation in this paper gives $L^e_{\mathrm{CPW}} = \Delta L_1 + \Delta L_2$.

IV. Conclusions

In this paper, we have described a simple concept of duality and based on which, find the quasi-static equivalent inductances of various CPW discontinuities via the computation of the equivalent capacitances of their complementary CPS discontinuities. Using the idea of excess charge and residual potential, we have applied the method of moments and Galerkin's approach to find the equivalent capacitances of CPS discontinuities. Comparison between the calculated results and those obtained by sophisticated full-wave analyses shows that the present quasi-static approximation is good even at the millimeter-wave spectrum. Since the present approach is simple, efficient, and feasible to deal with arbitrary shape of discontinuities, it is very suitable for the CAD in MMIC.

References

[1] R. H. Jansen, "Hybrid mode analysis of end effects of planar microwave and millimeterwave transmission lines," *IEEE Proc.*, vol. 128, pt. H, no. 2, pp. 77–86, Apr. 1981.

[2] R. W. Jackson, "Considerations in the use of coplanar waveguide for millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1450–1456, Dec. 1986.

[3] M. Drissi, V. F. Hanna, and J. Citerne, "Analysis of coplanar waveguide radiating end effects using the integral equation technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 112–116, Jan. 1991.

[4] N. I. Dib and P. B. Katehi, "Modeling of shielded CPW discontinuities using the space domain integral equation method (SDIE)," *J. Electromag. Waves Applicat.*, vol. 5, no. 4/5, pp. 503–523, Apr. 1991.

[5] G. Ghione and C. Naldi, "Analytical formulas for coplanar lines in hybrid and monolithic MICs," *Electron. Lett.*, vol. 20, pp. 179–181, Feb. 1984.

[6] R. N. Simons and G. E. Ponchak, "Modeling of some coplanar waveguide discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp. 1796–1803, Dec. 1988.

[7] M. Naghed and I. Wolff, "Equivalent capacitances of coplanar waveguide discontinuities and interdigitated capacitors using a three-dimensional finite difference method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-38, pp. 1808–1815, Dec. 1990.

[8] C. Sinclair and S. J. Nightingale, "An equivalent circuit model for the coplanar waveguide step discontinuity," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1992, pp. 1461–1464.

[9] R. Bromme and R. H. Jansen, "Systematic investigation of coplanar waveguide MIC/MMIC structures using a unified strip/slot 3D electromagnetic simulator," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1991, p. 1081–1084.

[10] M. Naghed, M. Rittweger, and I. Wolff, "A new method for the calculation of the equivalent inductances of coplanar waveguide discontinuities," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1991, pp. 747–750.

[11] A. F. Thomson and A. Gopinath, "Calculation of microstrip discontinuity inductances," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 648–655, Aug. 1975.

[12] P. Silvester and P. Benedek, "Equivalent capacitances of microstrip open circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 511–516, Aug. 1972.

[13] W. J. Getsinger, "Circuit duals on planar transmission media," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1983, pp. 154–156.

[14] R. F. Harrington, *Field Computation by Moment Methods.* New York: Macmillan, 1968, ch. 1.

[15] K. C. Gupta, R. Garg, and I. J. Bahl, *Microstrip Lines and Slotlines.* Dedham, MA: Artech, 1979, ch. 7.

[16] K. Beilenhoff, W. Heinrich, and H. L. Hartnagel, "Finite-difference analysis of open and short circuits in coplanar MMIC's including finite metallization thickness and mode conversion," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1992, pp. 103–106.

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